

The Importance of Elastic Anisotropy in Ultrasonic Inspection and Materials Characterization

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A Yenwong-Fai, L.P. Song, G. Amulele, Wits

Abstract

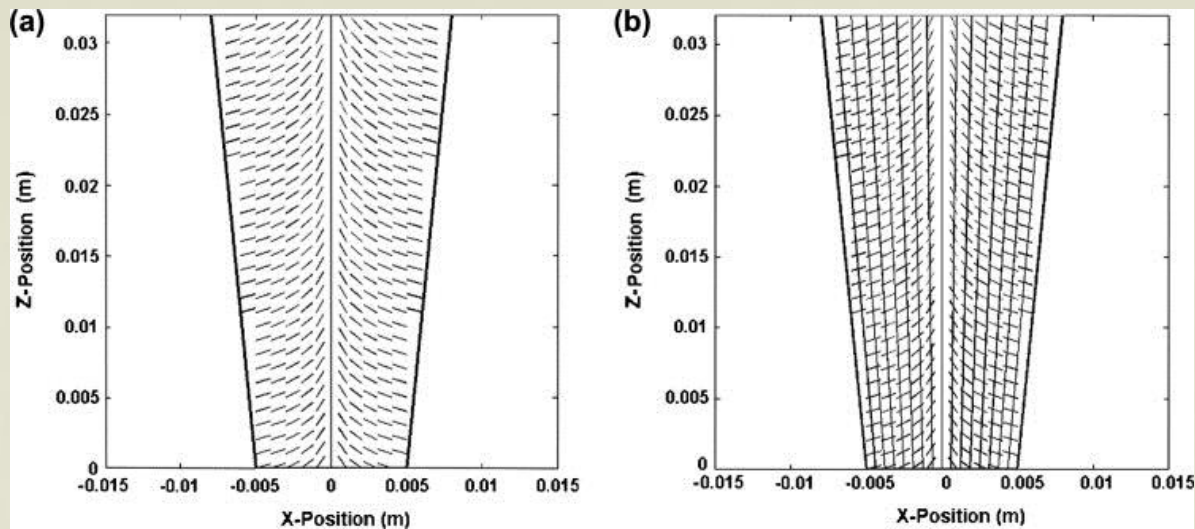
In this talk I address the question “Of what importance is elastic anisotropy in ultrasonic inspection and materials characterization? Is anisotropy something NDT professionals should be aware of, and are they likely to encounter it in their activities?” An elastically anisotropic medium is one in which properties such as elastic moduli and wave speed depend on direction. In the first part of the talk I will describe the principal types of anisotropic materials, crystals, fiber composites and so on. The second part of my talk will be to sketch some of the basic ideas that come up in the treatment of elastic wave propagation in anisotropic solids. The third part of the talk will deal with a number of applications that I have worked on, including phonon imaging, acoustic microscopy, laser ultrasound, surface acoustic waves and methods for characterizing sub-micron thick coatings.

Anisotropic Solids

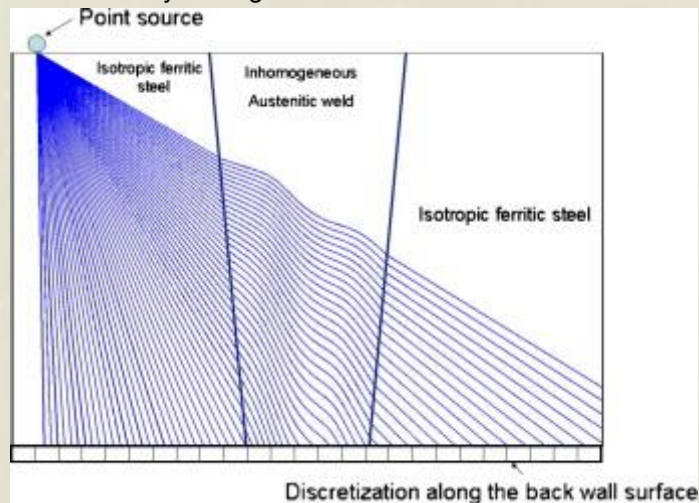
Anisotropy: properties such as wave speed depend on direction. Some types of anisotropic materials:

- **Crystals:** Semiconductors Si, Ge, GaAs, etc., Transducer materials quartz, LiNbO_3 , etc. Ni-based super-alloys, etc.
- **Polycrystalline textured materials:** Thin protective coatings, austenitic welds, etc.
- **Fiber composites:** C-fiber epoxy, glass fiber, etc.
- **Fibrous organic materials:** wood, bone, etc
- **Meta-materials:** superlattices, phononic crystals, oriented microfracture
- **Stressed solids:** elastoacoustic effect
- **Geological structures:** Stratification, tectonic effects

A.G. Every et al., Wave Motion **50**, 1197 (2013).



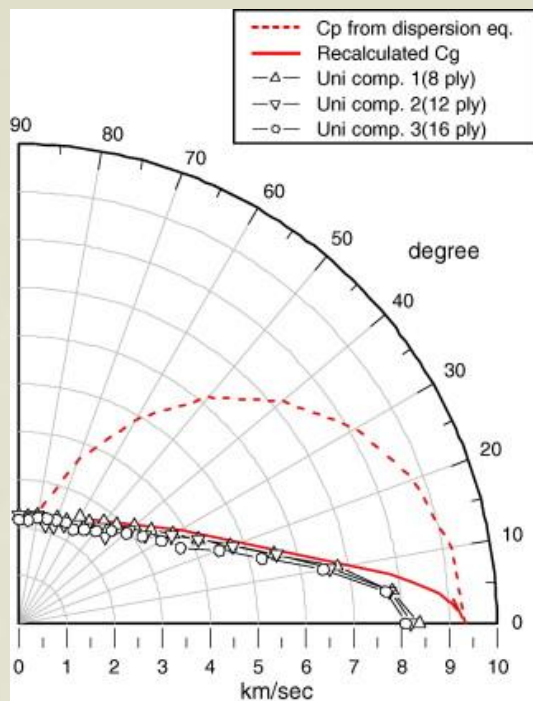
(a) Inhomogeneous weld structure, (b) layered representation of inhomogeneous weld. Weld boundary inclination angle = 5.36° . $T = 0.54$, and $\eta = 1$ in Eq. (1). Illustration of ray tracing below.



S.R. Kolkoori, M.-U. Rahman, P.K. Chinta, M. Ktreutzbruck, M. Rethmeier, J. Prager

Ultrasonic field profile evaluation in acoustically inhomogeneous anisotropic materials using 2D ray tracing model: Numerical and experimental comparison

Ultrasonics, Volume 53, Issue 2, 2013, 396–411



Phase velocity (C_p), theoretically recalculated group velocity (C_g), and experimentally measured group velocity curve for a unidirectional composite plate.

Sang-Ho Rhee, Jeong-Ki Lee, Jung-Ju Lee

The group velocity variation of Lamb wave in fiber reinforced composite plate

Ultrasonics, Volume 47, Issues 1–4, 2007, 55–63

W. Sachse, A.G. Every and M. O. Thompson
Impact of laser pulses on composite materials,
ASME, AMD-Vol. 116, 51-62 (1990)

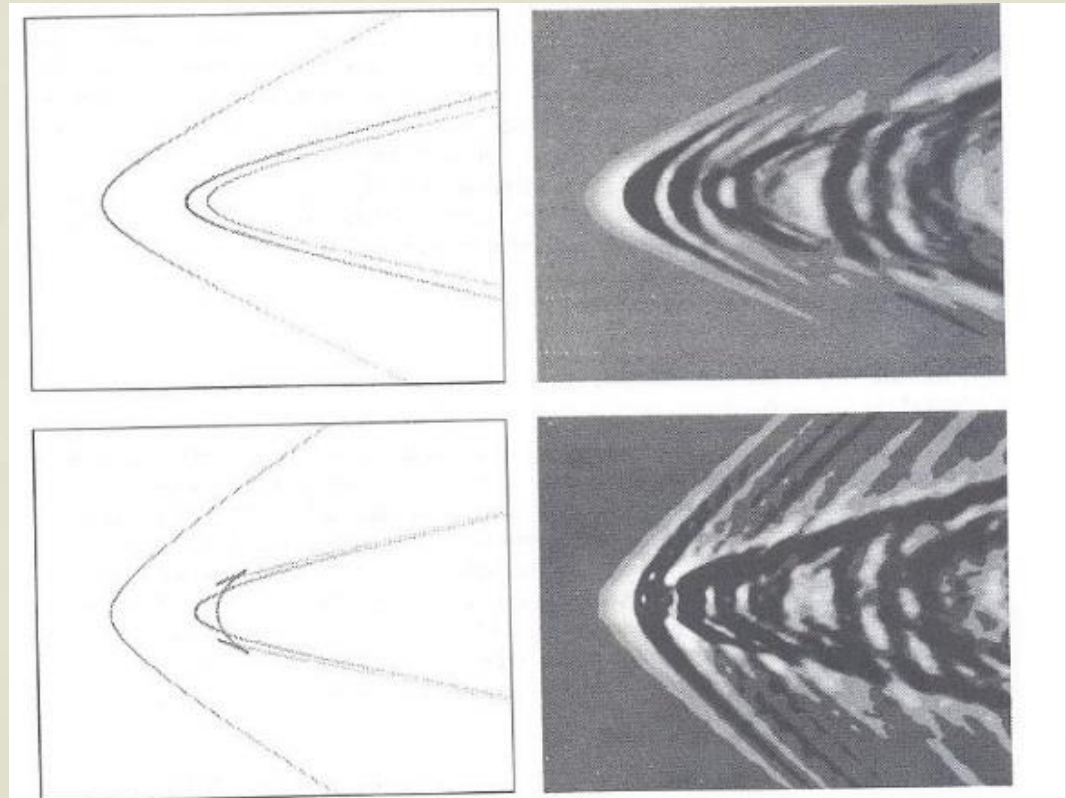
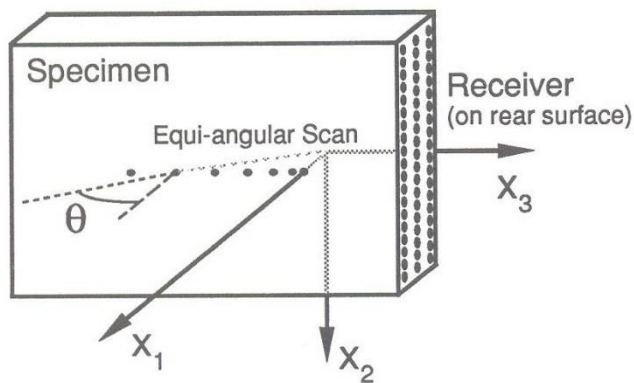


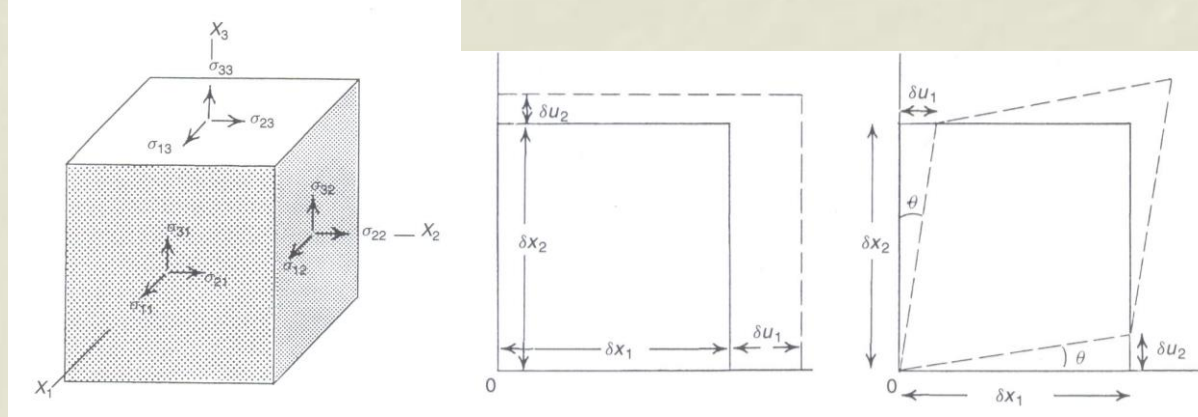
Figure 3: Computed and measured scan for a uni-directional fiberglass/polyester specimen. (Top) Scan perpendicular to the fiber axes; (Bottom) Scan parallel to the fiber axes.

Elasticity of Anisotropic Solids - an Introduction

We deal with general states of elastic stress and strain defined in the following way:

Stress Tensor $\sigma_{ij} = \lim_{\delta A \rightarrow 0} \frac{\delta F_i^j}{\delta A}$; $i, j = 1, 2, 3$,

where δF_i^j is the i 'th component of the force acting on the cube across the surface facing outwards along the x_j Cartesian direction, and δA is the area of that face



Strain Tensor $\epsilon_{kl} = \frac{1}{2} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right)$.

where $\mathbf{u}(\mathbf{x}) = \{u_i(\mathbf{x})\}$ is the displacement field, and $\mathbf{x} = (x_1, x_2, x_3)$ is the undisturbed particle position.

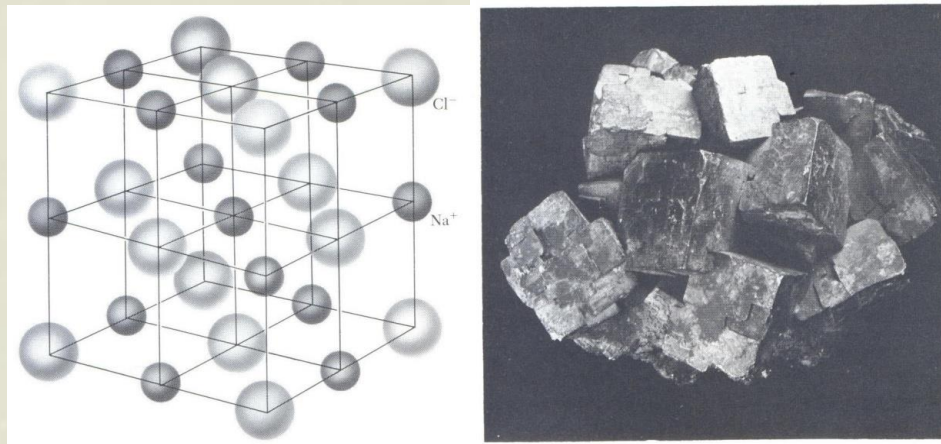
Since $\sigma_{ij} = \sigma_{ji}$ and $\epsilon_{kl} = \epsilon_{lk}$, there are thus only six independent components of stress and of strain, in contracted notation $\sigma_\alpha; \epsilon_\beta$, $\alpha, \beta = 1, \dots, 6$.

Hooke's Law for an ideal linear elastic solid $\sigma_{ij} = \sum_{k,l=1,3} c_{ijkl} \epsilon_{kl}$; $\sigma_\alpha = \sum_{\beta=1,6} C_{\alpha\beta} \epsilon_\beta$.

The coefficients $c_{ijkl}; C_{\alpha\beta}$ constitute the **elastic stiffness tensor/matrix** of the solid. Because $C_{\alpha\beta} = C_{\beta\alpha}$, there at most 21 independent elastic constants $C_{\alpha\beta}$.

Simplifications Resulting from Materials Symmetries

Anisotropic solids, with few exceptions, possess some degree of structural symmetry, such as 2-, 3-, 4- or 6-fold axes of rotational symmetry, reflection planes and inversional symmetry. Their physical properties, and in the case of crystals their habit, reflect this symmetry.



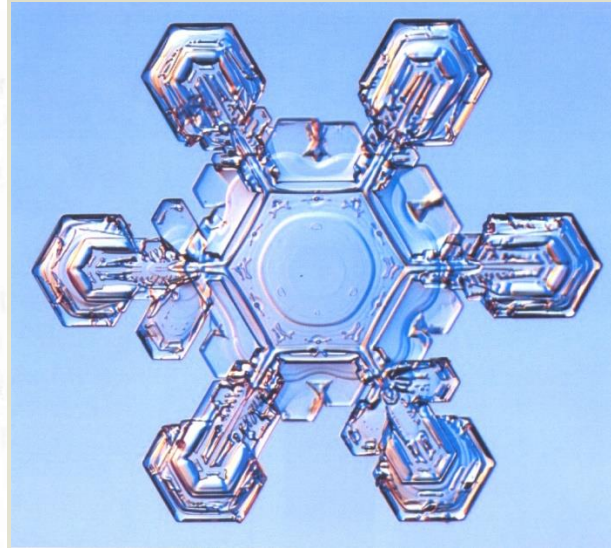
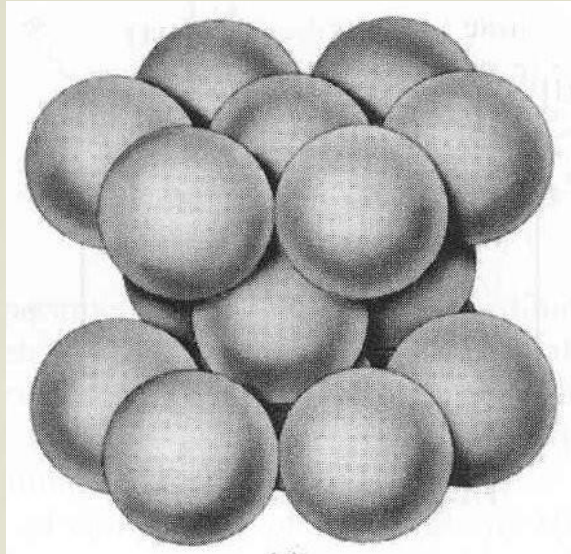
Cubic crystals NaCl crystal structure and PbS crystals.

There are only 3 independent moduli $C_{11} = c_{1111}$, $C_{12} = c_{1122}$, $C_{44} = c_{2323}$.

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}$$

For isotropy $C_{44} = (C_{11} - C_{12})/2$, so only 2 independent moduli

$$C_{11} = \lambda + 2\mu \text{ and } C_{44} = \mu.$$



Hexagonal crystals, e.g. close packed metals, Ice - a snow flake.
There are only 5 independent moduli.

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C' \end{bmatrix}, \quad C' = (C_{11} - C_{12}) / 2$$

Transverse isotropy, e.g. uniaxial fibre composites, laminated solids, has the same form of matrix. Thus hexagonal crystals are transversely isotropic as regards linear elasticity.

Wave Equation for an Anisotropic Solid and its Solutions

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sum_{j,k,l} c_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} \quad ; i = 1, 2, 3 .$$

The wave equation admits plane wave solutions of the form

$$u_i = U_i \exp[i(\mathbf{k} \cdot \mathbf{x} - \omega t)],$$

where the **unit polarization vector** $\mathbf{U} = (U_i)$, the **wave vector** $\mathbf{k} = (k_i)$, ($|\mathbf{k}| = 2\pi / \lambda$, where λ = wavelength, and direction normal to the phase fronts) and the angular frequency ω , are subject to the set of three equations

$$(c_{ijkl} k_j k_l - \rho \omega^2 \delta_{ik}) U_k = 0, \quad i = 1, 2, 3,$$

known as the **Christoffel equations**. The corresponding **secular equation** is

$$D(\mathbf{k}, \omega) = |c_{ijkl} k_j k_l - \rho \omega^2 \delta_{ik}| = 0.$$

By dividing all terms by k^2 one obtains

$$|c_{ijkl} n_j n_l - \rho v^2 \delta_{ik}| = 0,$$

where $\mathbf{n} = \mathbf{k} / k$ is the wave normal, and $v = \omega / k$ is the **phase velocity**. For given $\mathbf{n}$ there are three solutions for v^2 and the corresponding $\mathbf{U}$ are mutually perpendicular, one corresponding to a quasi-longitudinal wave, and the other two to quasi-transverse waves.

In Seismology anisotropy tends to be a small effect and can be treated as a perturbation



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Linearized approximations for phase velocities of elastic waves in weakly anisotropic media

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J. Phys. D: Appl. Phys. **33** (2000) L81–L85. Printed in the UK

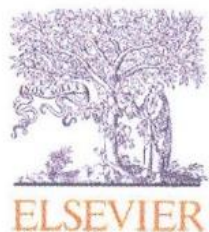
PII: S0022-3727(00)14896-X

RAPID COMMUNICATION

Approximate formulae for acoustic wave group slownesses in weakly orthorhombic media

Lin-Ping Song and A G Every

Department of Physics, University of the Witwatersrand, PO Wits 2050, South Africa



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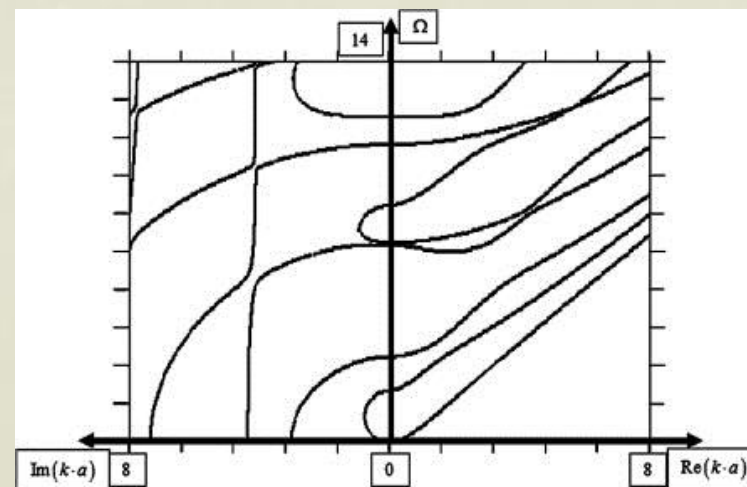
Analysis of non-axisymmetric wave propagation in a homogeneous piezoelectric solid circular cylinder of transversely isotropic material

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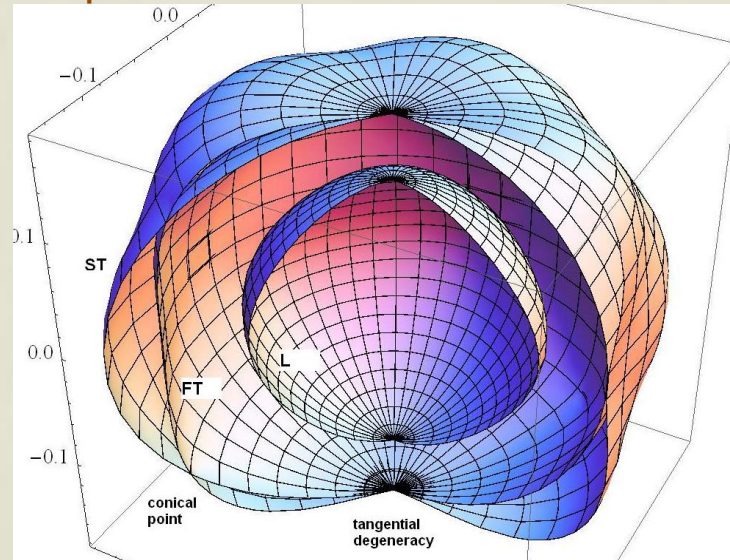


The Slowness Surface

The slowness surface plays a central role in the analysis of many wave phenomena, including reflection and transmission at surfaces, surface waves, energy flux concentration, etc. It is a centro-symmetric surface of three sheets representing the directional dependence of the acoustic slowness or inverse phase velocity $s = 1/v$, and as such it is the locus of slowness vectors $\mathbf{s} = \mathbf{k} / \omega = \mathbf{n} / v$. The slowness surface is identical in shape to the constant frequency surfaces, differing only in scale. The **equation for the slowness surface** is obtained from the Christoffel secular equation by dividing by ω^6 :

$$S(\mathbf{s}) = |c_{ijkl}s_js_l - \rho\delta_{ik}| = 0,$$

and is of degree 6 in the components of $\mathbf{s}$.



Slowness surface of silicon crystal, depicting real solutions of the slowness equation. It is a centrosymmetric surface of three sheets

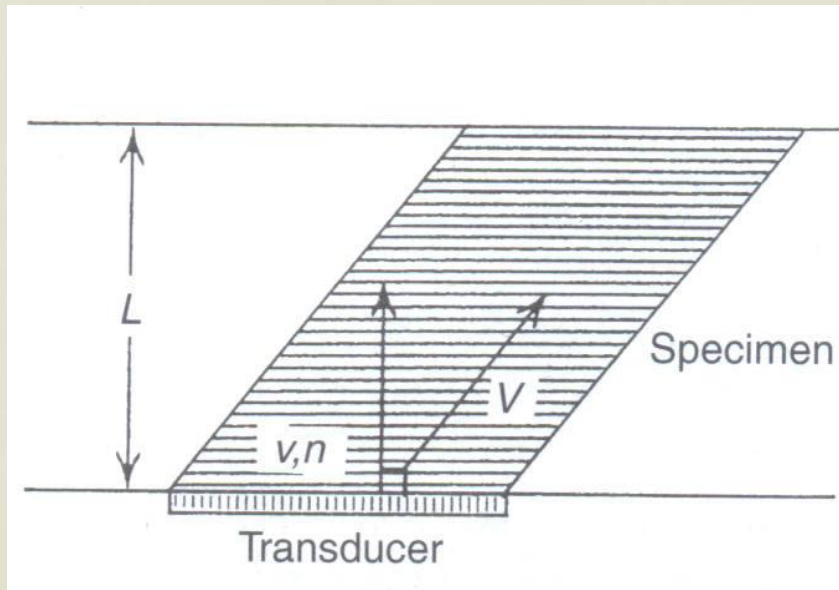
Group/Ray Velocity

The velocity characterizing signal (wave packet) propagation is the group or ray velocity. In the absence of dispersion and attenuation it is the same as the velocity with which the energy of a plane homogeneous wave propagates. It is given by

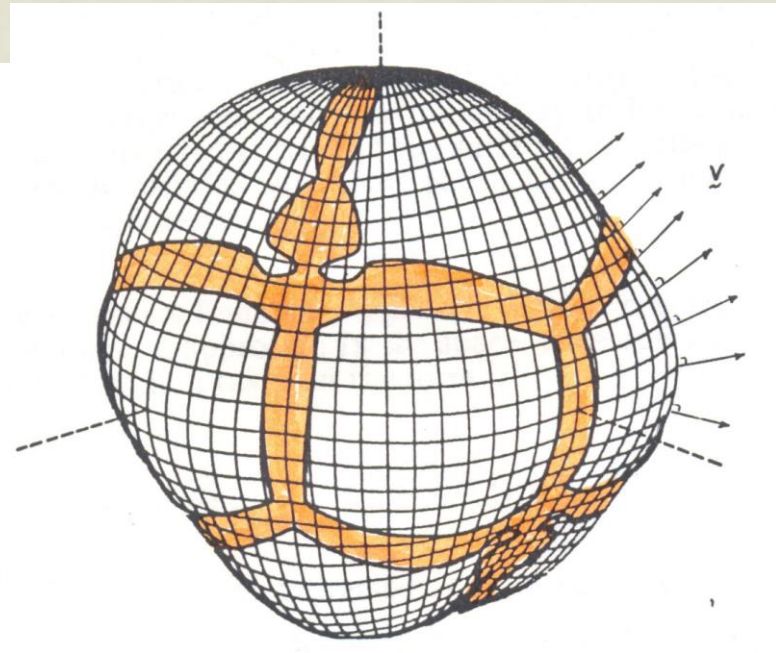
$$\mathbf{V} = \left(\frac{\partial \omega}{\partial k_1}, \frac{\partial \omega}{\partial k_2}, \frac{\partial \omega}{\partial k_3} \right) = \nabla_{\mathbf{k}} \omega(\mathbf{k}).$$

It follows from the defining equation that the group velocity is normal to the constant frequency and slowness surfaces.

The group velocity can deviate significantly in direction from the phase velocity, in direction and magnitude, with some striking consequences.



Phase and group velocity.

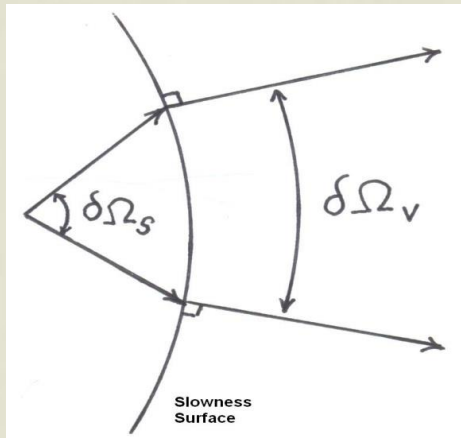


ST Slowness surface and rays for sapphire

Energy Focusing - Phonon Focusing

Consider emission from a point source, of incoherent sound with an omnidirectional distribution of $\mathbf{k}$ ($\mathbf{s}$) vectors, which we treat in the context of the ray approximation.

Diagram shows a cone of slowness vectors of solid angle $\delta\Omega_s$, the energy flux of which is radiated into solid angle $\delta\Omega_v$ subtended by the corresponding ray vectors $\mathbf{V}$.



The phonon (energy flux) enhancement factor is defined as $A = |\delta\Omega_s / \delta\Omega_v|$

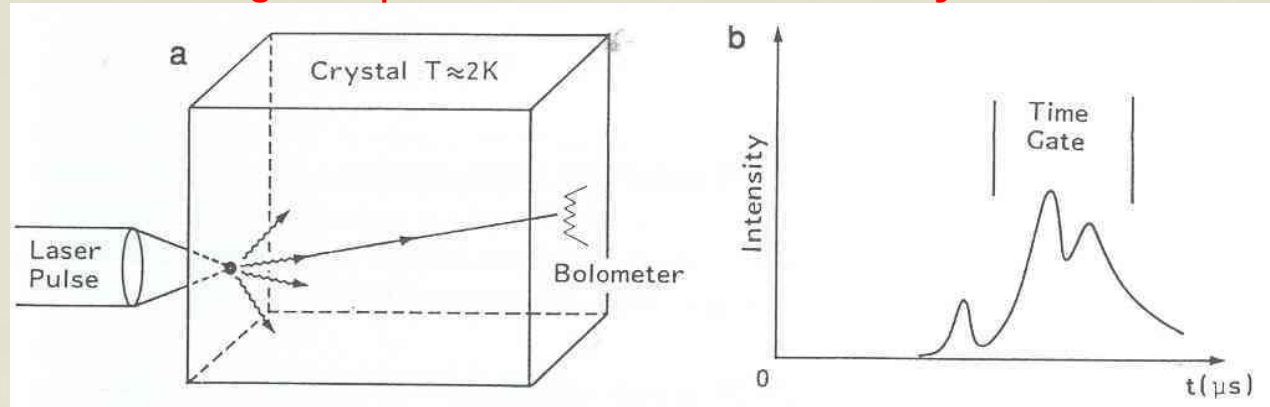
It follows that $A = |s^3 V K|^{-1}$; $K = L_1 L_2$

where K is the Gaussian curvature and L_1 and L_2 are the principal curvatures of the slowness surface. A is infinite for $K = 0$, a caustic.

A.G. Every, Phys.Rev. **B24**, 3456 (1981)

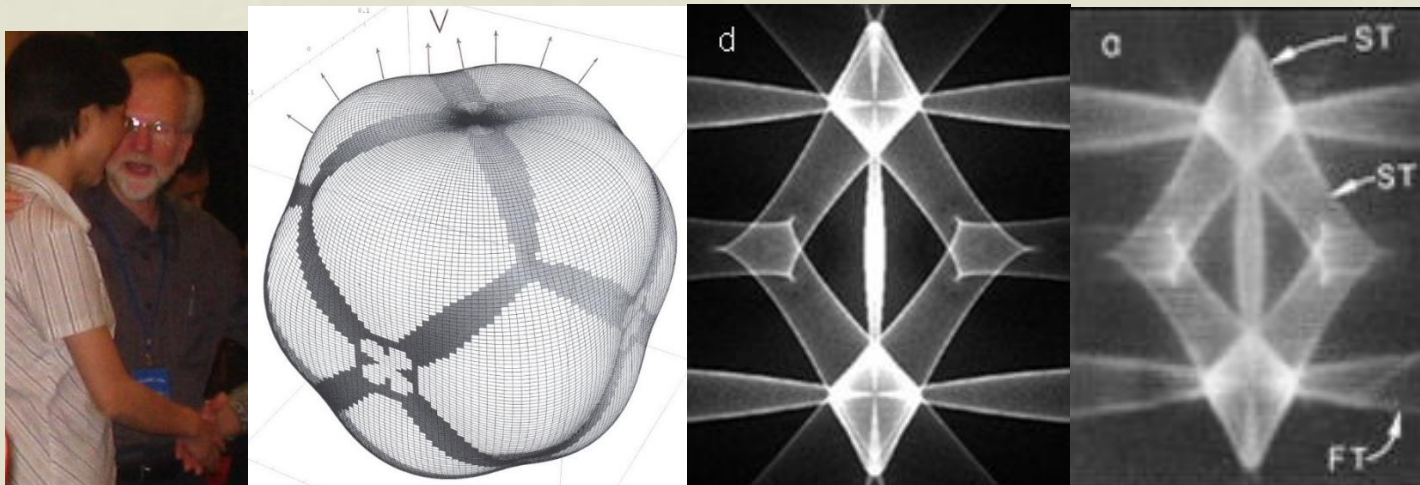
Phonon Imaging

Heat resides in the thermal vibrations (very high frequency acoustic waves $\sim 10^{11}\text{Hz}$). At high T there is considerable phonon scattering and heat flow is diffusive. At low T there is little scattering and phonons travel **ballistically** and exhibit strong **focusing**.



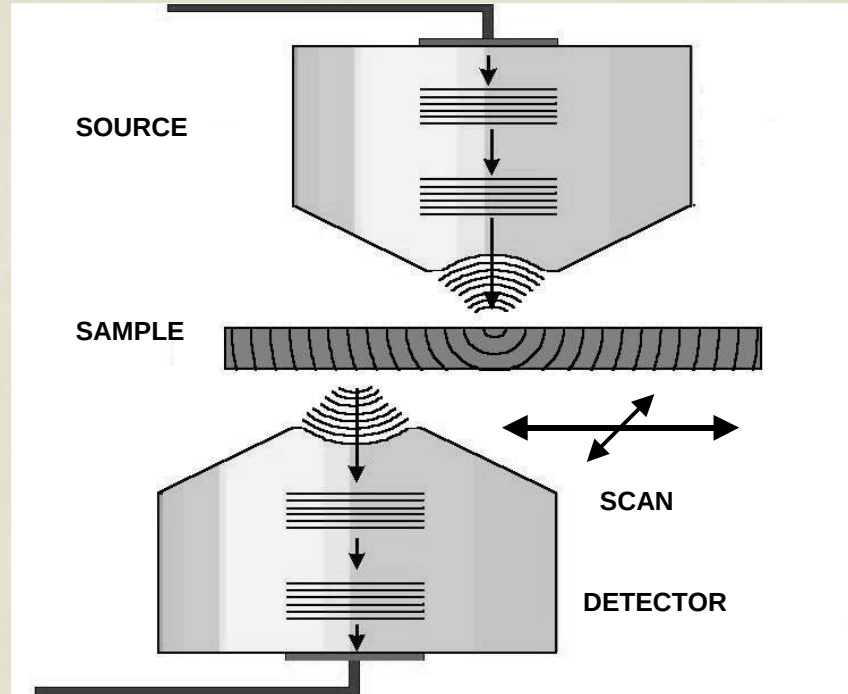
(a) Principle of phonon imaging, (b) Typical ballistic heat pulse profile

J.P. Wolfe, Imaging Phonons, CUP (1998). A.G. Every, PR B34, 2852 (1986).



Slowness surface of GaAs and calculated and measured phonon image of GaAs(110)

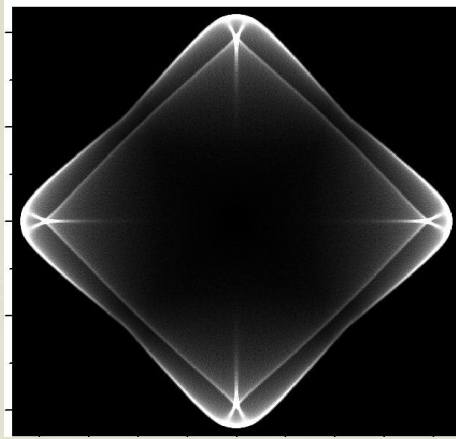
Displacement response $G_{ij}(\mathbf{x}, \omega)$ of an Infinite Anisotropic Solid to a Time Harmonic Point Force. Collaboration with M. Pluta and W. Grill



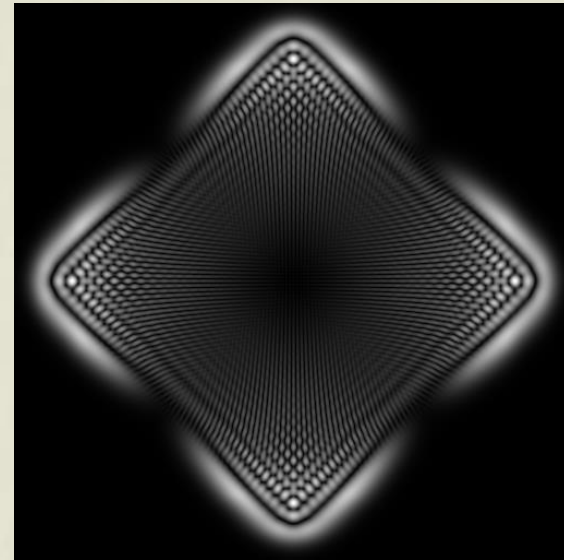
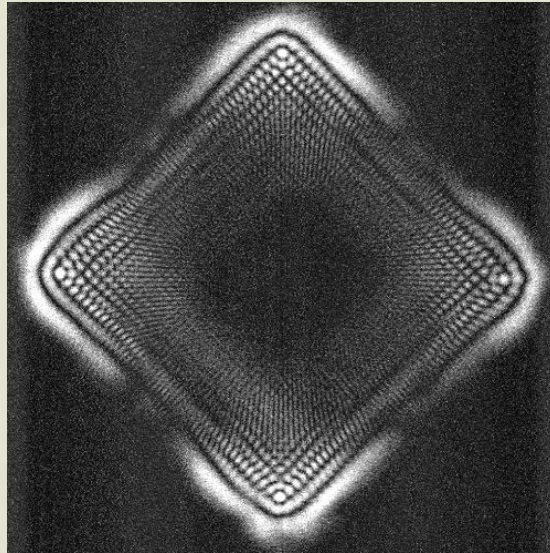
Schematic representation of transmission acoustic microscopy setup for measuring $G_{33}(\mathbf{x}, \omega)$

*M. Pluta, A.G. Every and W. Grill, Phys. Rev. **B67**, 094117 (2003).*

By raster scanning source or detector, two-dimensional phase and amplitude images are acquired.



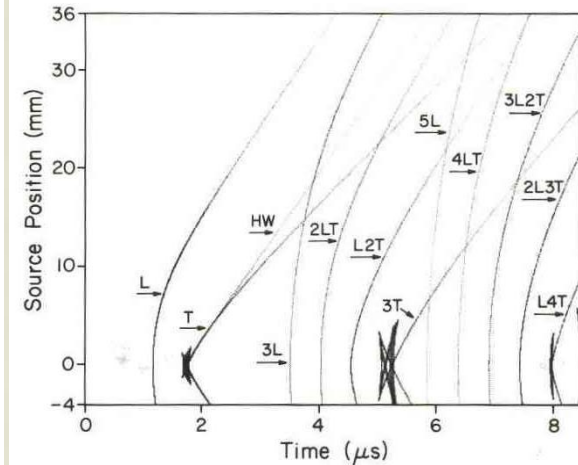
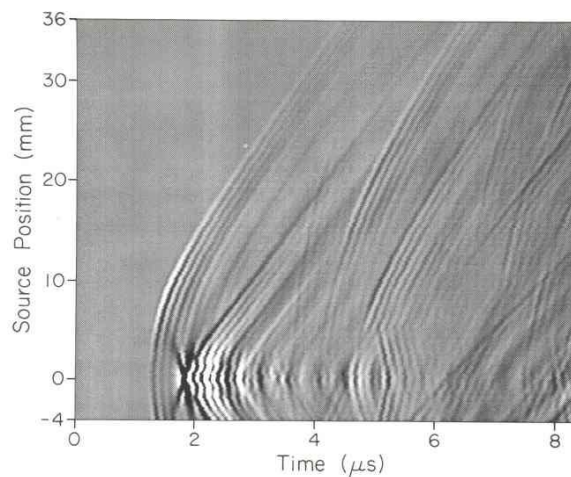
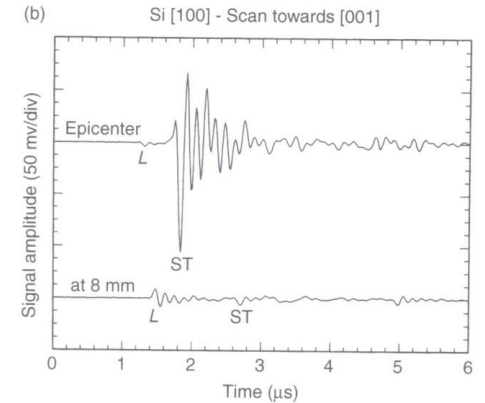
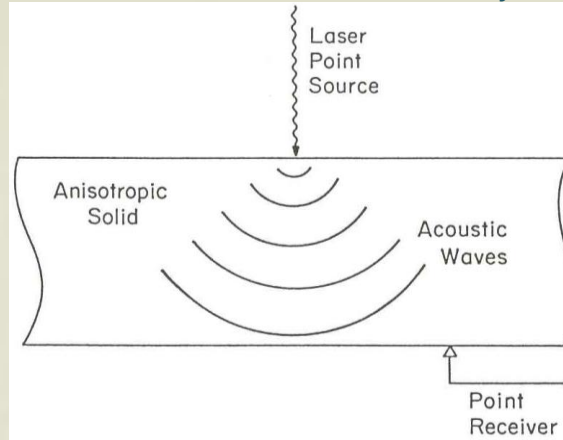
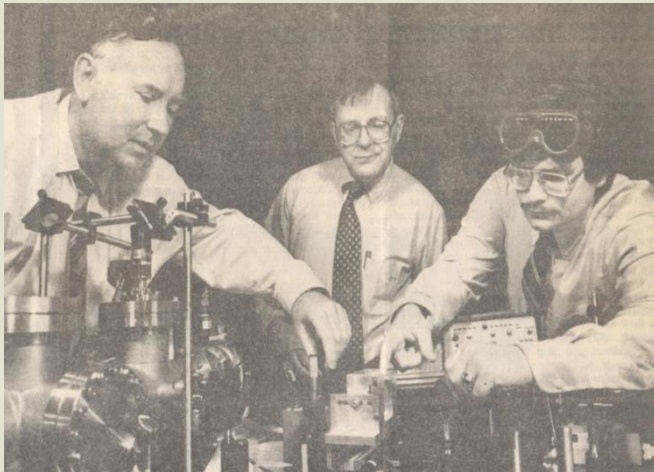
Focusing pattern of GaAs(001)



Measured and calculated TAM amplitude images of a 4.8 mm thick, (100) oriented GaAs crystal for $f = 362$ MHz (Pluta, Every & Grill) and optimised elastic constants.

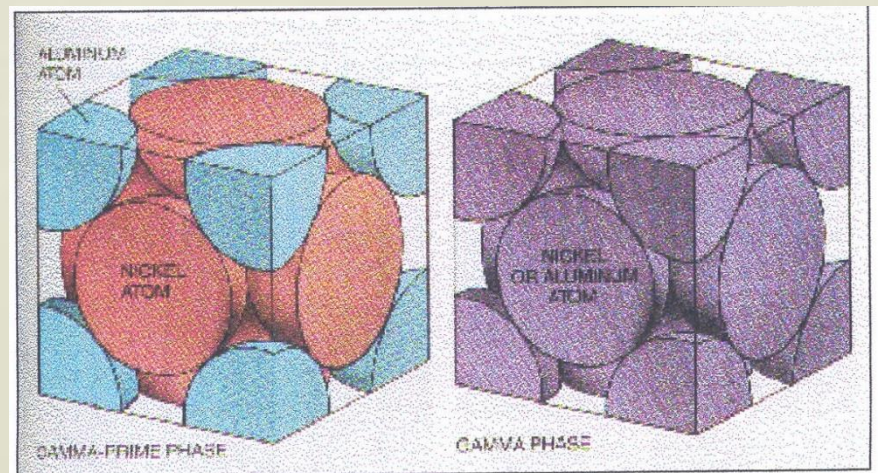
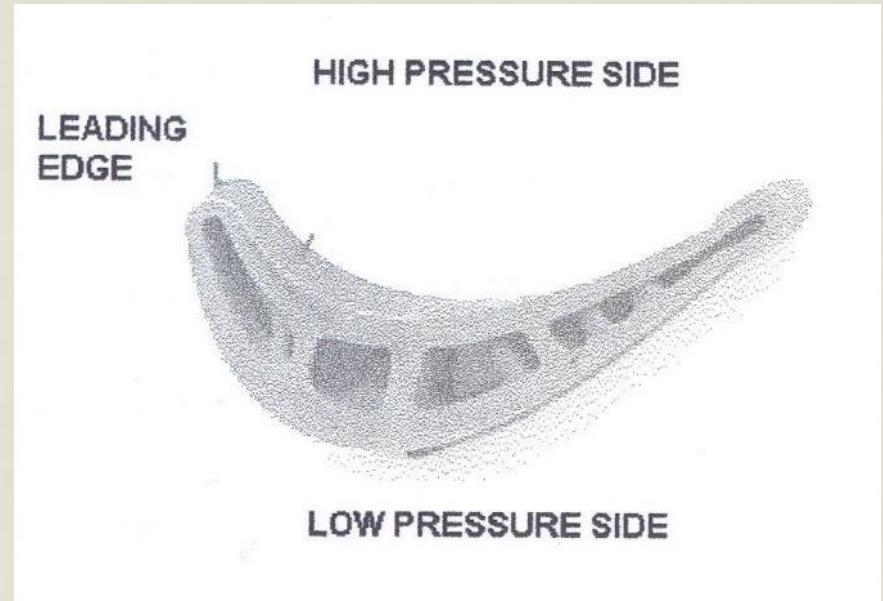
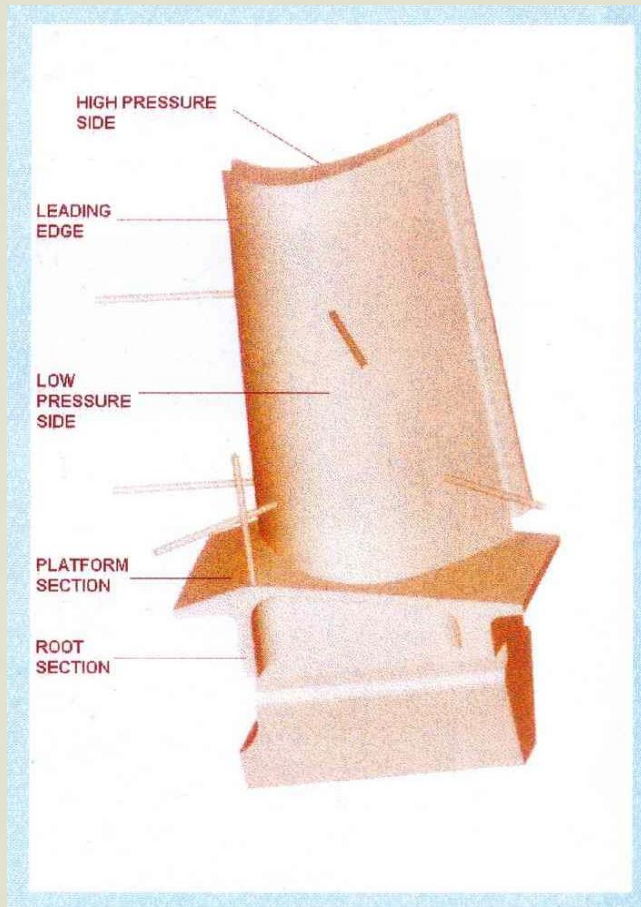
Laser generated Ultrasound Scans in Si(001)

AG Every and W Sachse, *Imaging of Laser-Generated Ultrasonic Waves in Silicon*, Phys. Rev B44, 6689 (1991)



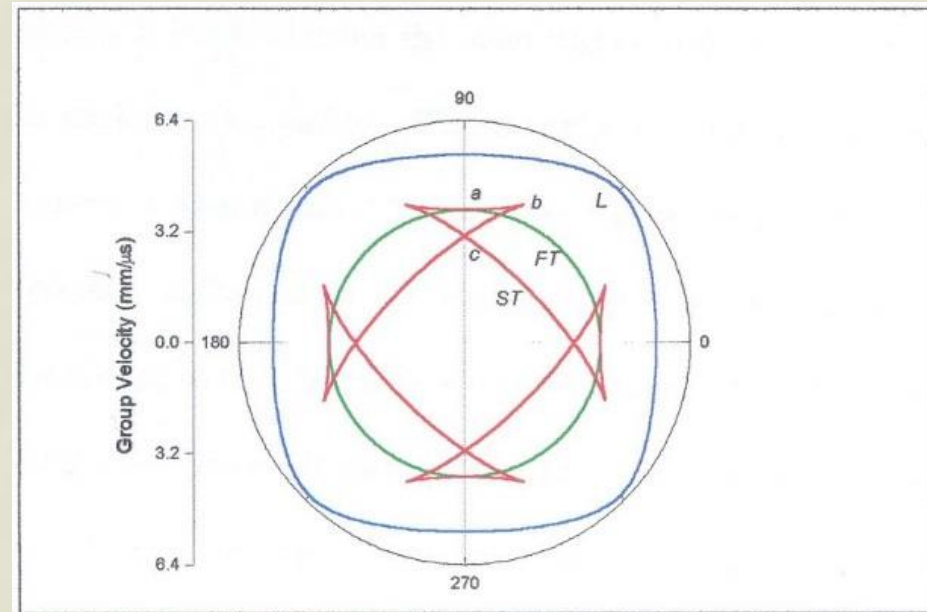
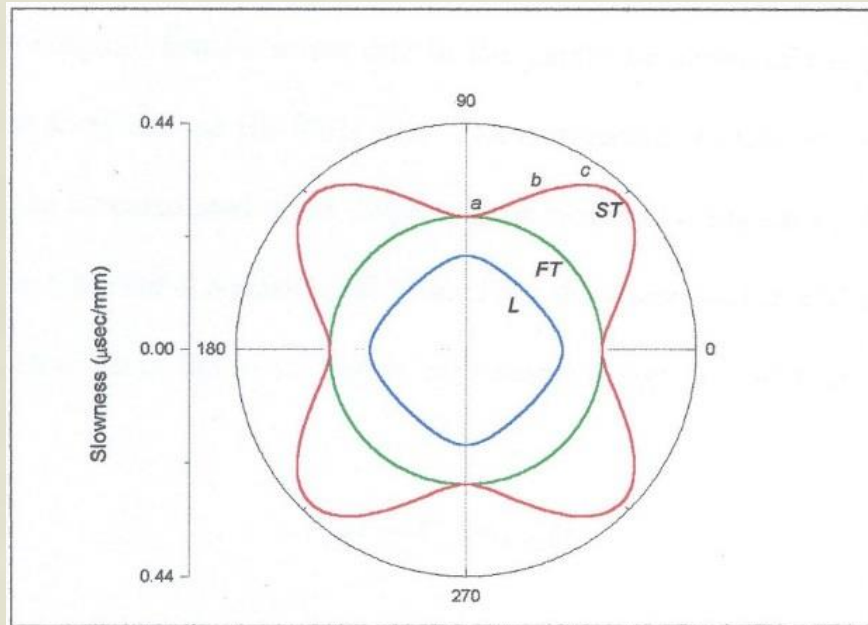
Transducer ringing is initiated by wave arrivals travelling at group velocity $\mathbf{V}$. The excitation, whether thermoelastic or ablative, is radially symmetric, and so for [100] scan only L and ST waves are observed. Elastic constants of Si Recovered from $\mathbf{V}$'s by two stage optimization process.

Single Crystal Nickel-based Super-alloy Gas Turbine Blade

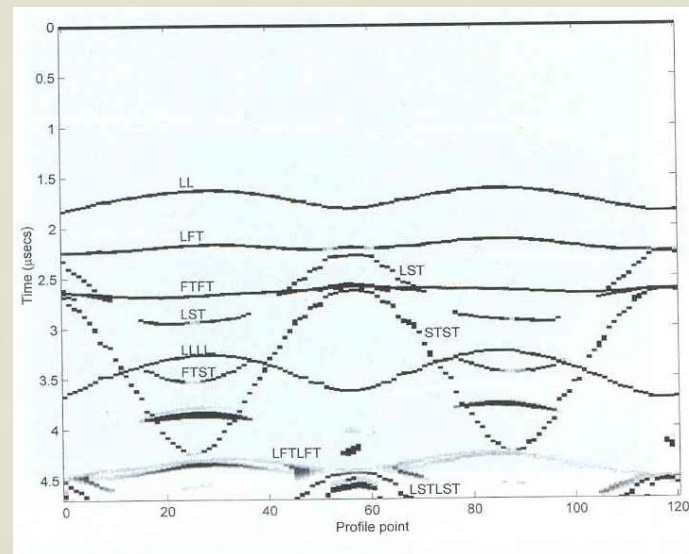
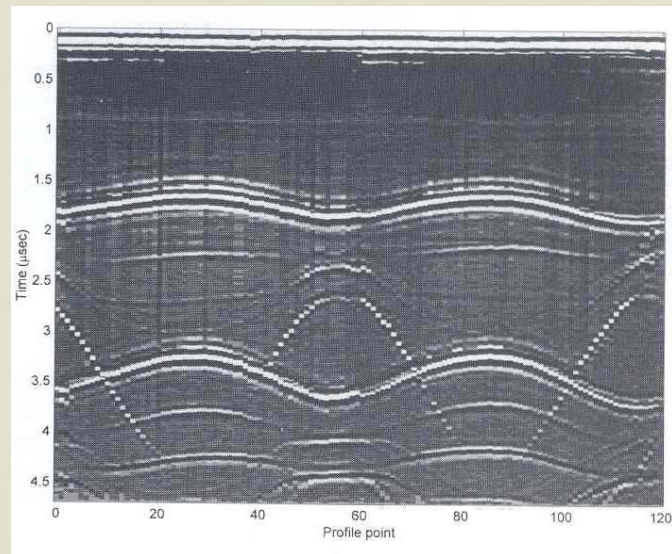
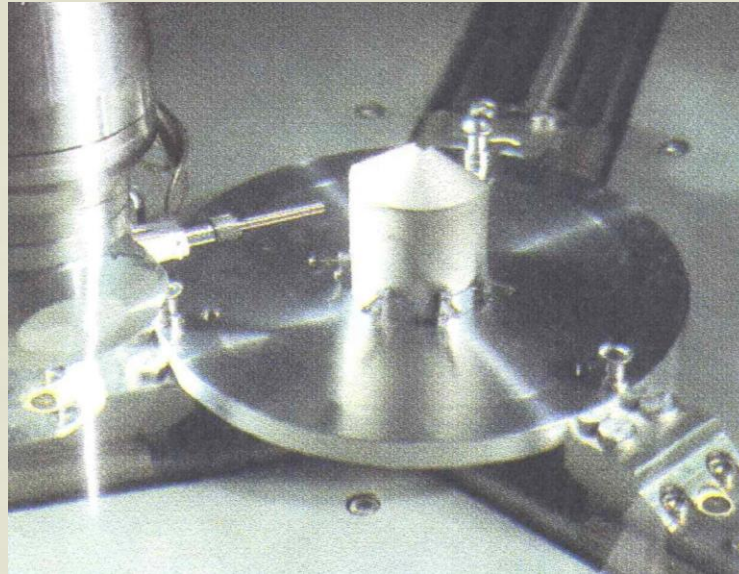


G. Amulele and A.G. Every, Ultrasonics **38**, 252 (2000);
IEEE Trans. Ultrason. Ferro. Freq. Contr. **49**, 307 (2002);
Rev. Prog. QNDE **18**, 2001 (1999).

Cube Plane Sections of Slowness and Group Velocity Surfaces of Nickel-based Super-alloy CMSX-4

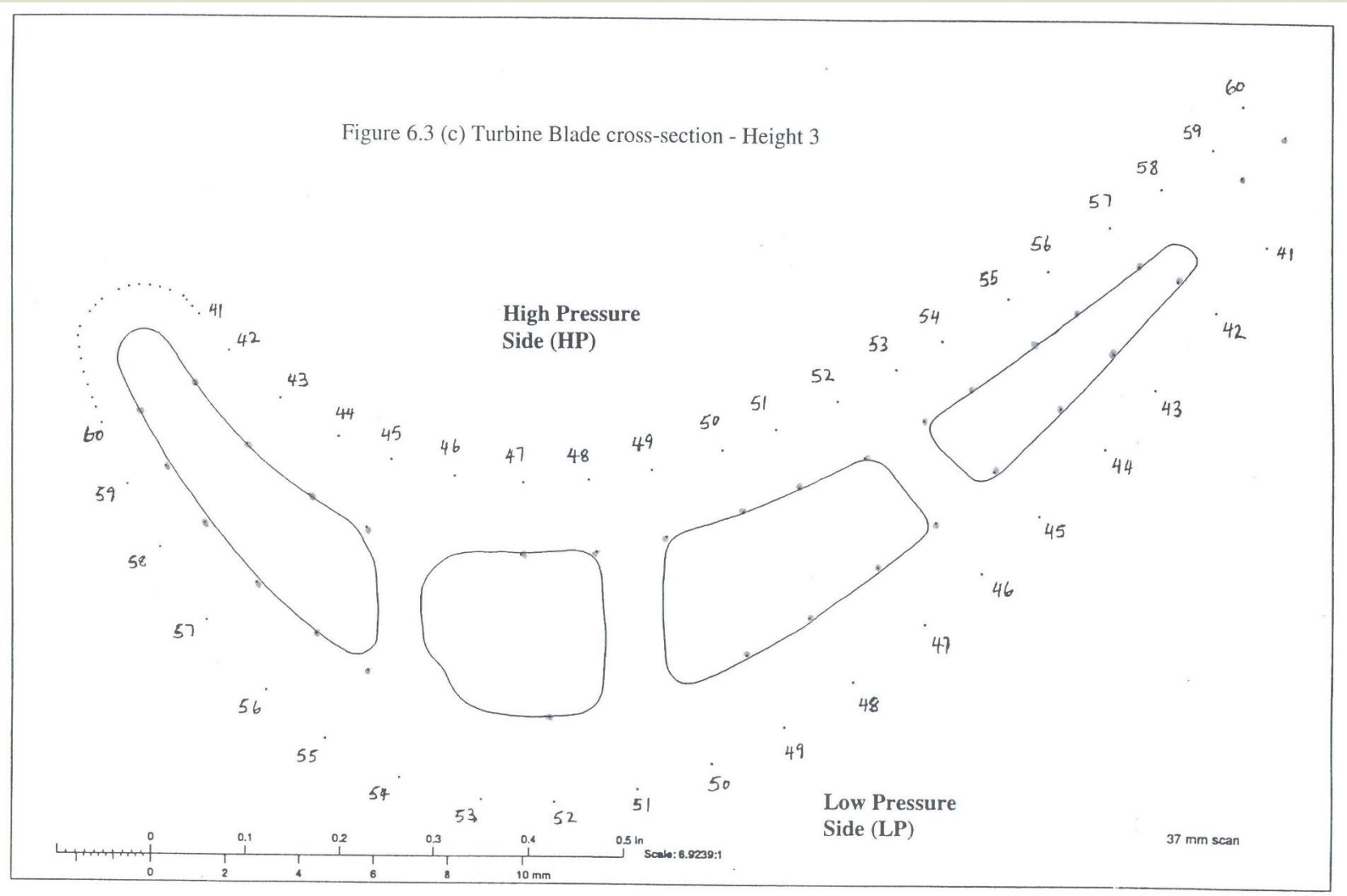


Focused Probe Ultrasonic Scans of Single Crystal Superalloy Components



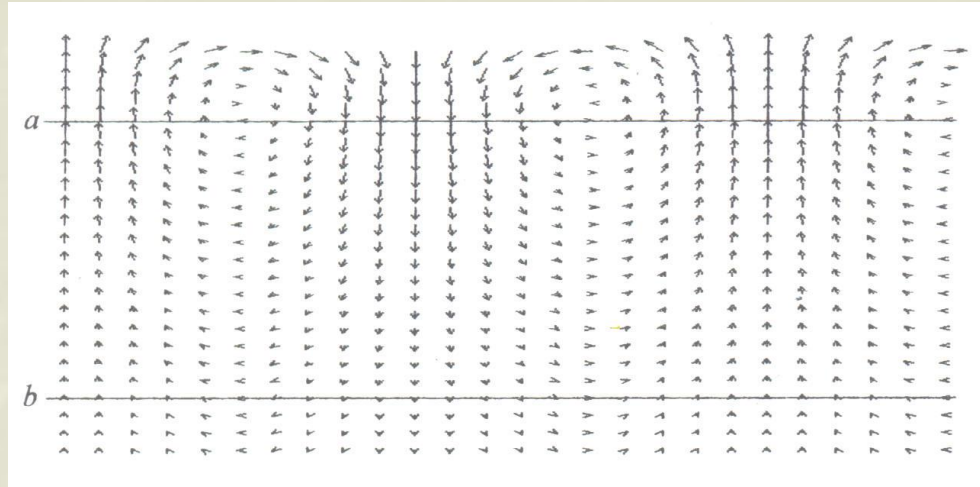
Scan over cylindrical section and ray model calculation (Every and Amulele)

Measurements on turbine blade at numbered points on outer surface. Inferred position of inner surface indicated by dots. Blade was then sectioned and inner surface determined by coordinate measuring machine, as indicated by lines.



Surface and Interfacial Waves

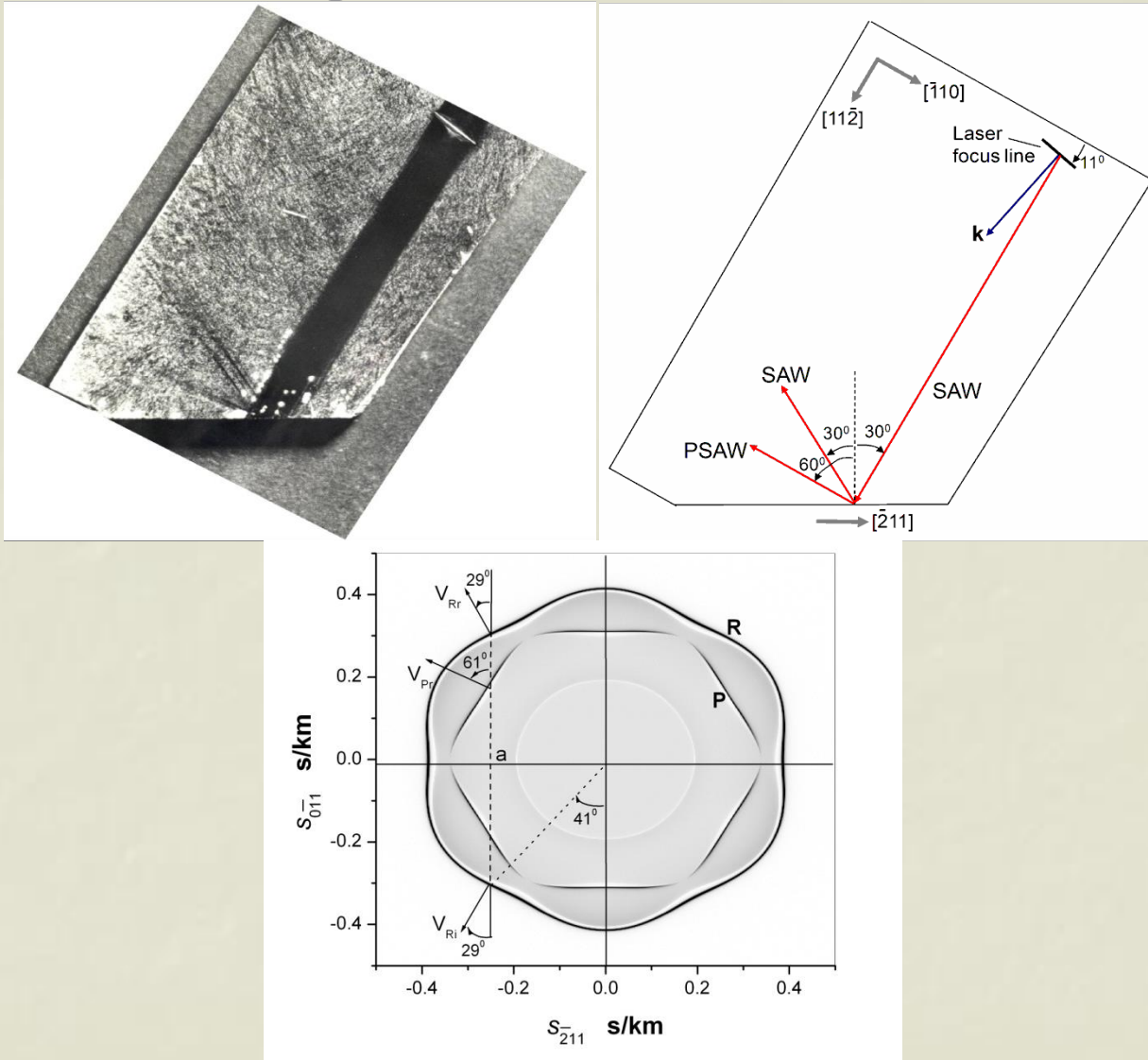
A **Rayleigh surface acoustic wave (SAW)** is a superposition of 3 evanescent waves that satisfies the boundary conditions. The displacement field is confined near the surface and travels along the surface with slowness $s_{||}$



For an isotropic solid, a particle at the surface executes **retrograde elliptical motion**. For anisotropic solids the particle motion is not confined to the sagittal plane, the slowness varies with direction and the group velocity of the Rayleigh wave deviates from its phase velocity

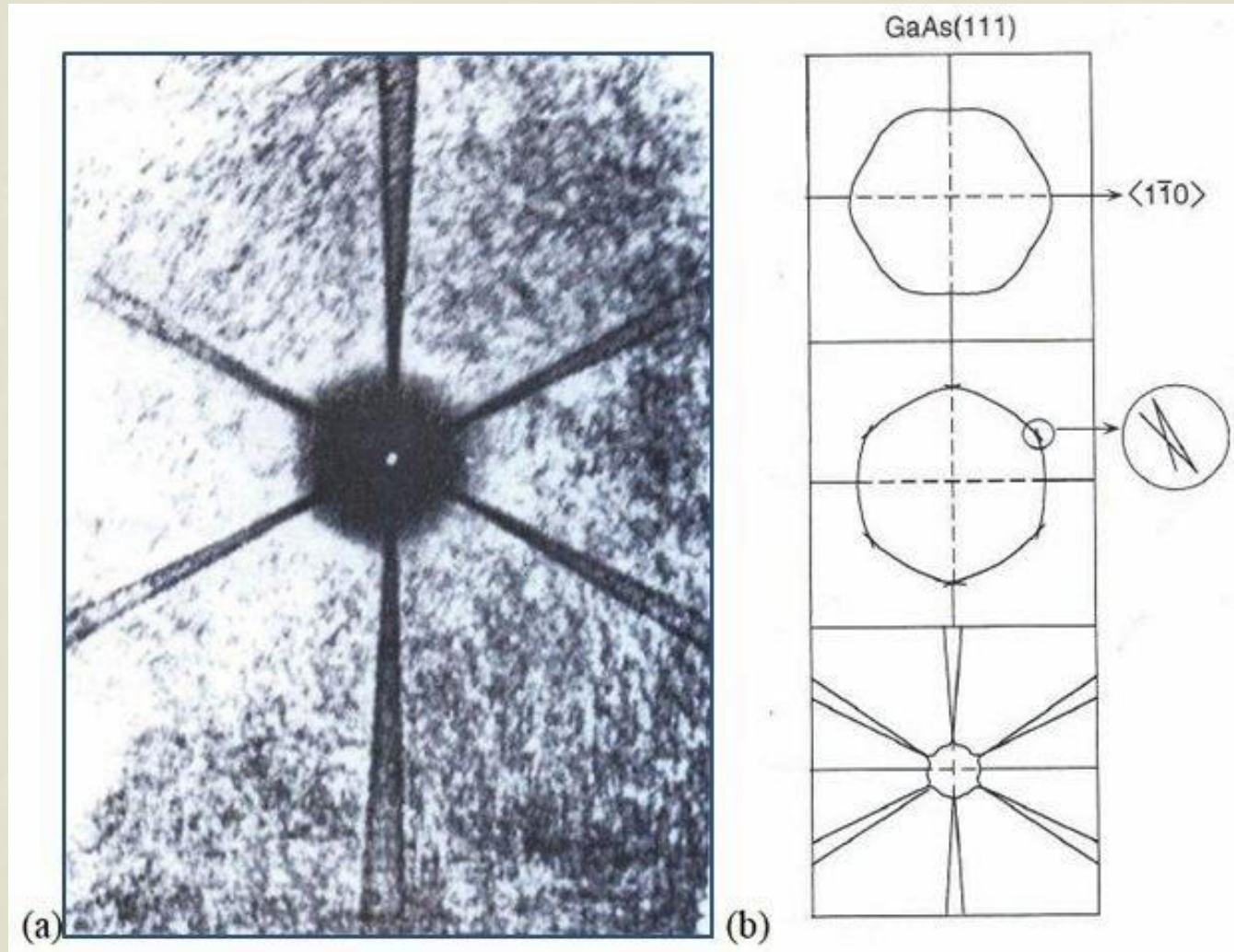
There exist **pseudo-surface waves**, waves with a small partial wave component, which is radiative, and causes the surface wave to attenuate with distance.

"Bi-refringence" of Laser-generated Surface Waves on GaAs Crystal



A. G. Every and A. A. Maznev, *Ray Splitting in the Reflection and Refraction of Surface Acoustic Waves in Anisotropic Solids*, J. Acoust. Soc. Am. 127 (5), 2813-2818 (2010).

Powder Detachment Method for Observing Anisotropy Focusing of Laser-generated SAW on GaAs



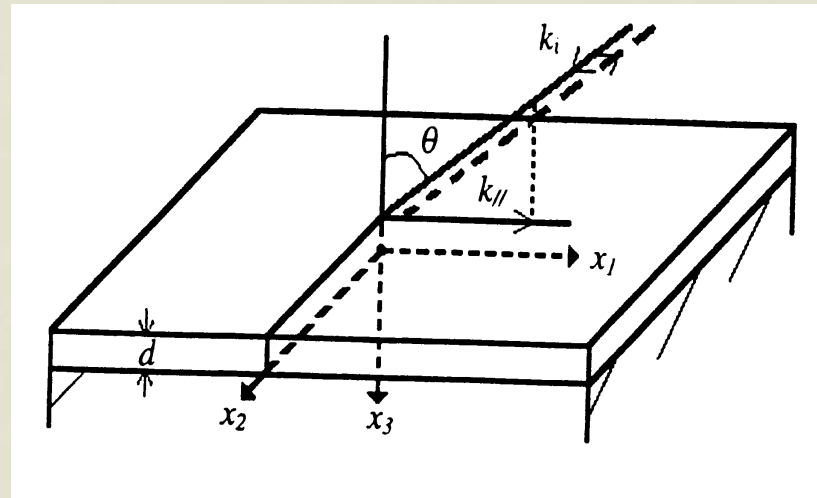
Kolomenskii and Maznev, Phys. Rev. B **48**, 14502 (1993). Maznev and Every, Acta Acustica **1**, 137 (1993).

Surface Brillouin Scattering (SBS) Collaboration with J. D. Comins

In SBS a laser beam is incident on a surface, and the spectrum of the light scattered by dynamic fluctuations in the surface profile is determined. The kinematic conditions for back-scattering from surface corrugations of velocity v are

$$k_{//} = 2k_i \sin \theta, \quad k_i = \text{laser wave vector, and frequency shift } \omega = \pm v k_{//}$$

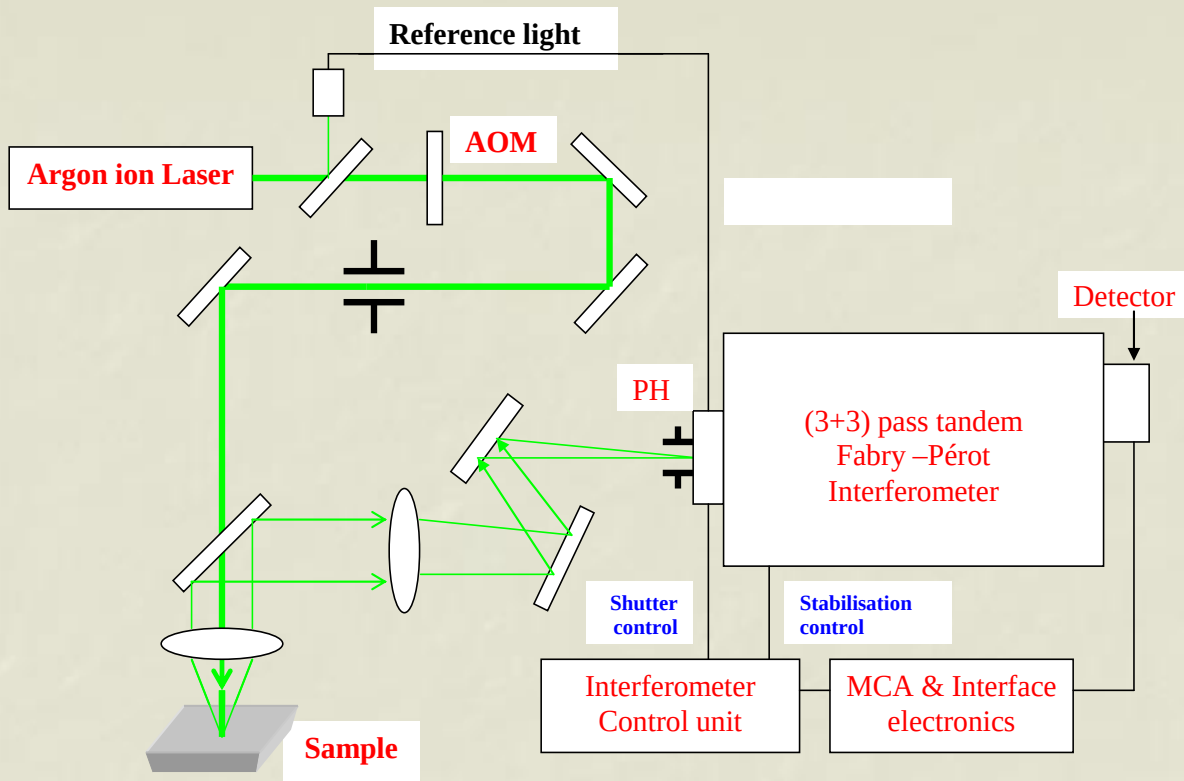
The scattering cross section is given by $\frac{d^2 \sigma}{d\Omega d\omega} = \frac{AT}{\omega} \text{Im} G_{33}(k_{//}, \omega)$



SAW gives rise to sharp line in the spectrum, bulk acoustic waves to a continuum. In partially transparent materials light is scattered by dynamic fluctuations in the strain field. Frequency shifts for surface wave scattering (up to ~30 GHz).

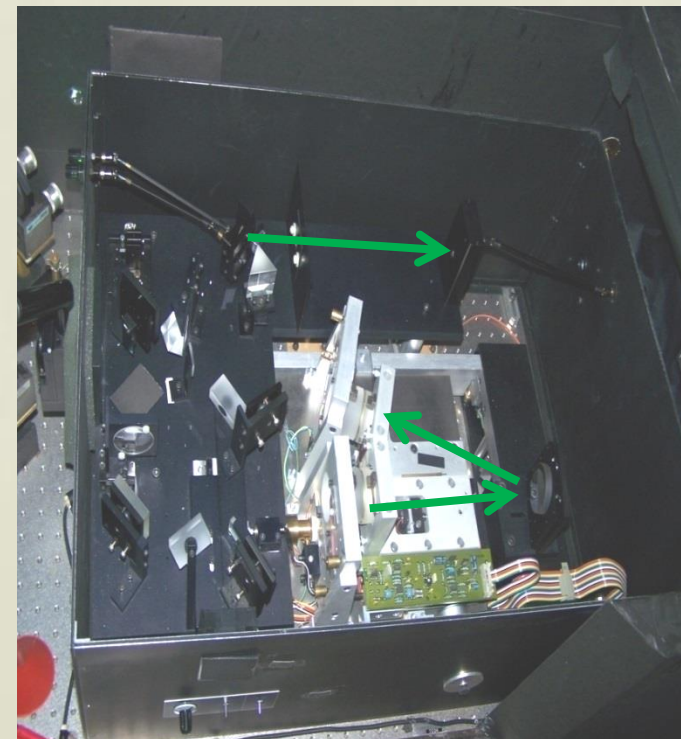
Review: M. G. Beghi, A. G. Every, V. Prakapenka and P. V. Zinin, in *Ultras. Electromag. NDE for Struc. and Mat. Characterization*, T. Kundu ed., CRC Press, (2012), Chap. 10.

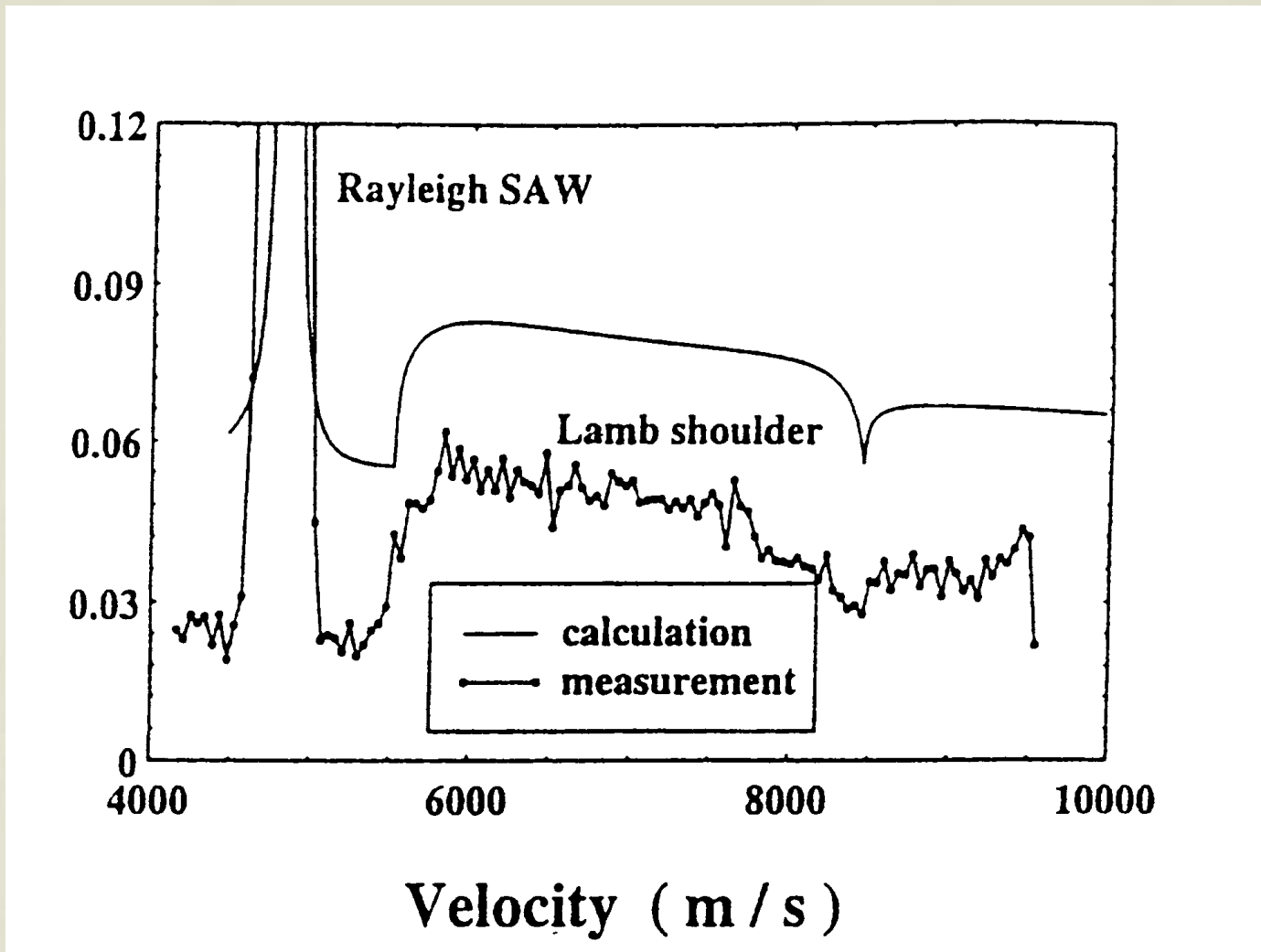
SBS Experimental Arrangement



Argon ion laser $\lambda = 514.5 \text{ nm}$
Detector: Low noise silicon avalanche diode ($< 1 \text{ c/s}$)
Contrast: $\sim 10^{11}$ obtained by multi-passing

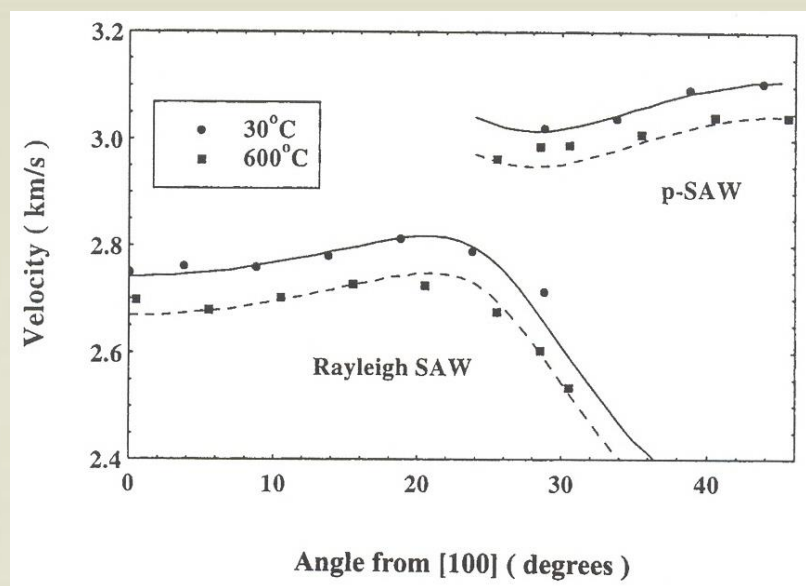
Aerial view - (3+3)
pass tandem Fabry-
Pérot interferometer



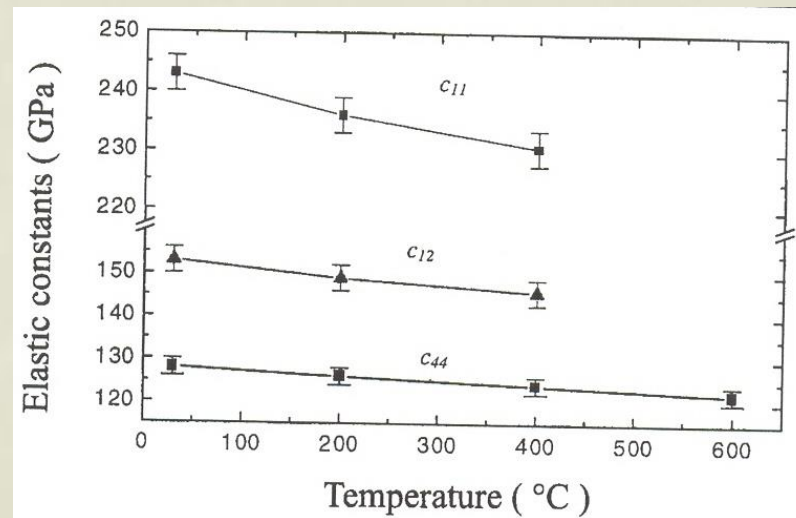


SBS spectrum of $VC_{0.75}$. The calculated spectrum is proportional to $\text{Im} G_{33}(k_{//}, \omega)$

Xinyia Zhang, J.D. Comins, A. G. Every and P.R.Stoddart, Int. J. Ref. Metals and Hard Mat. **16**, 303 (1998)
 $C_{11}=440$ GPa, $C_{12}=92$ GPa, $C_{44}=136$ GPa



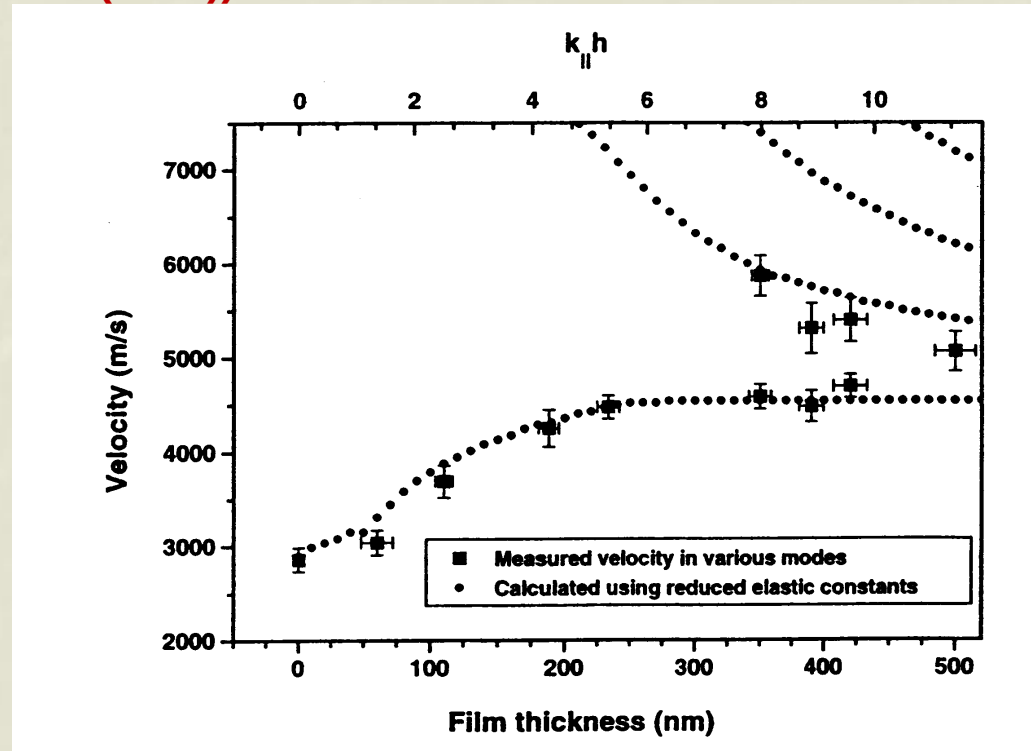
Directional dependence of the Rayleigh and pSAW velocities for Ni-based superalloy CMSX-4 (X. Zhang, P.R. Stoddart, J.D. Comins and A.G. Every, J. Phys.: Condens. Matter 13, 2281 (2001).).



Temperature variation of elastic constants inferred from RW, pSAW and lateral wave velocities.

Guided Surface Modes for the Fast-on-slow Combination TiN Coating on High Speed Steel

(Wenjun Pang, A. G. Every, J. D. Comins, P. R. Stoddart and X. Zhang, J. Appl. Phys. 86, 311 (1999))



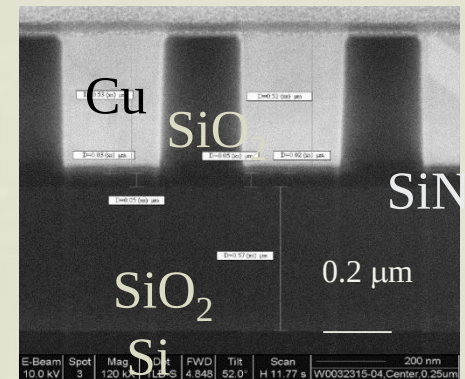
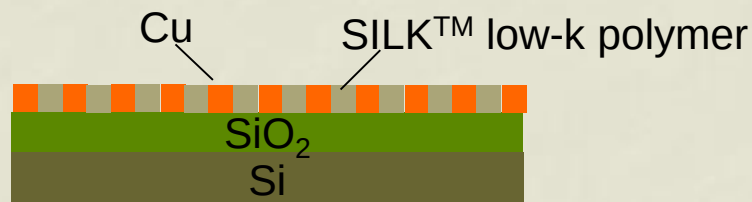
Fit to measured dispersion using adjusted elastic constants of TiN

At a critical value of $k_{\parallel} h$, the Rayleigh SAW degenerates into the HSS continuum.

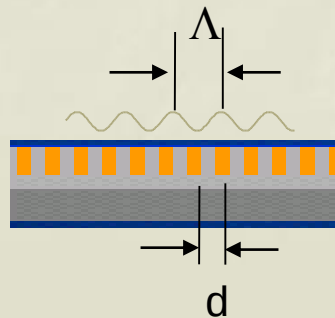
For large $k_{\parallel} h$ the lowest resonance evolves into the Rayleigh wave for TiN.

Periodically Patterned Thin Film Structures Encountered in the Semiconductor Industry

Collaboration with Alex Maznev, MIT

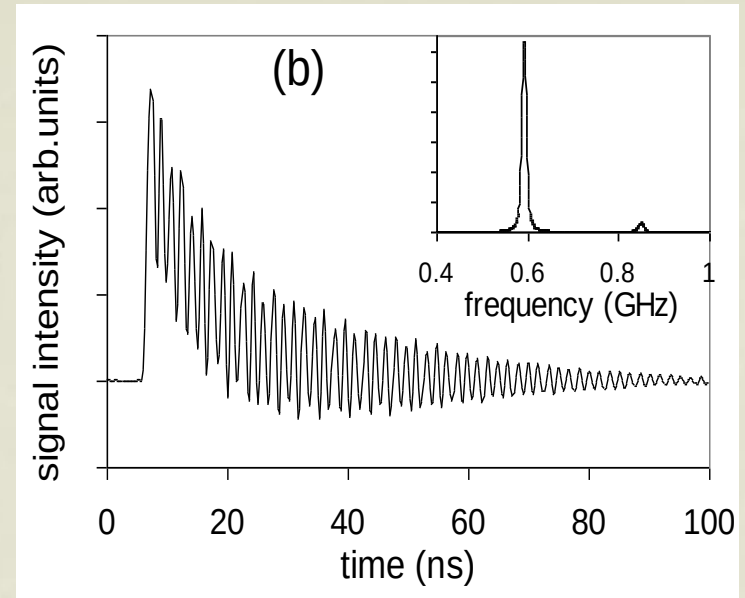
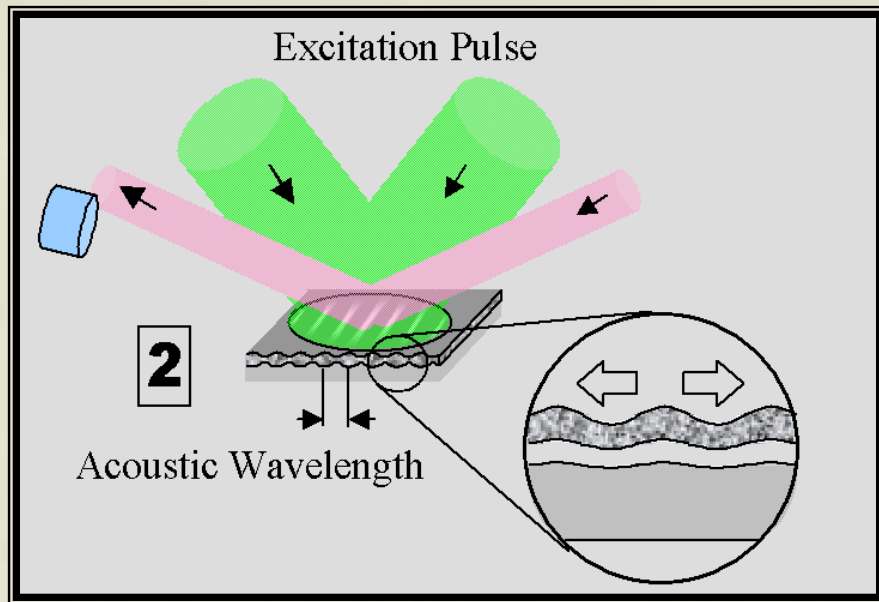


Characterization with SAW



$\Lambda \sim d$ State-of-the-art $d=130$ nm
How to model?

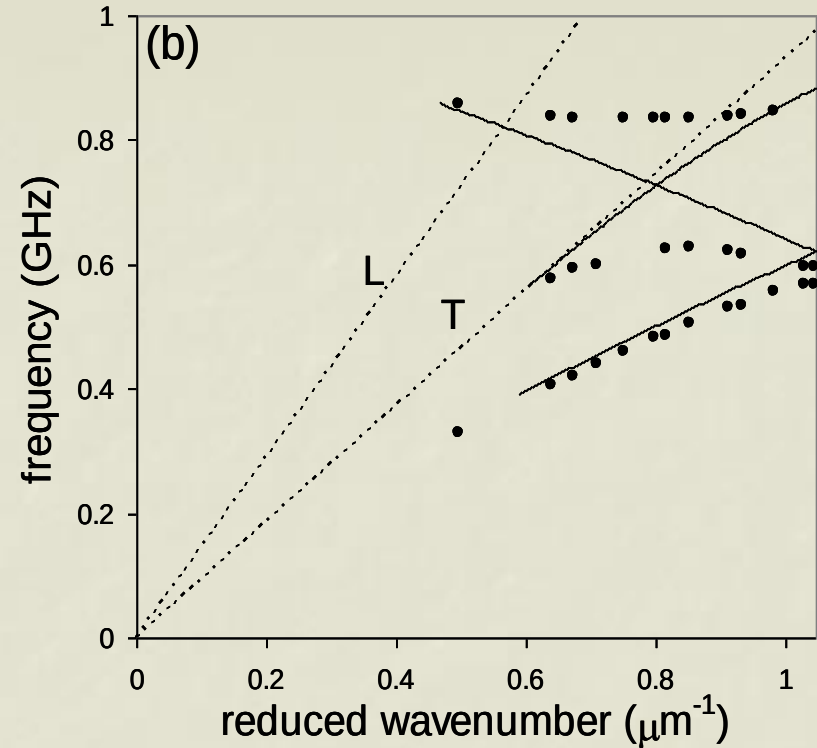
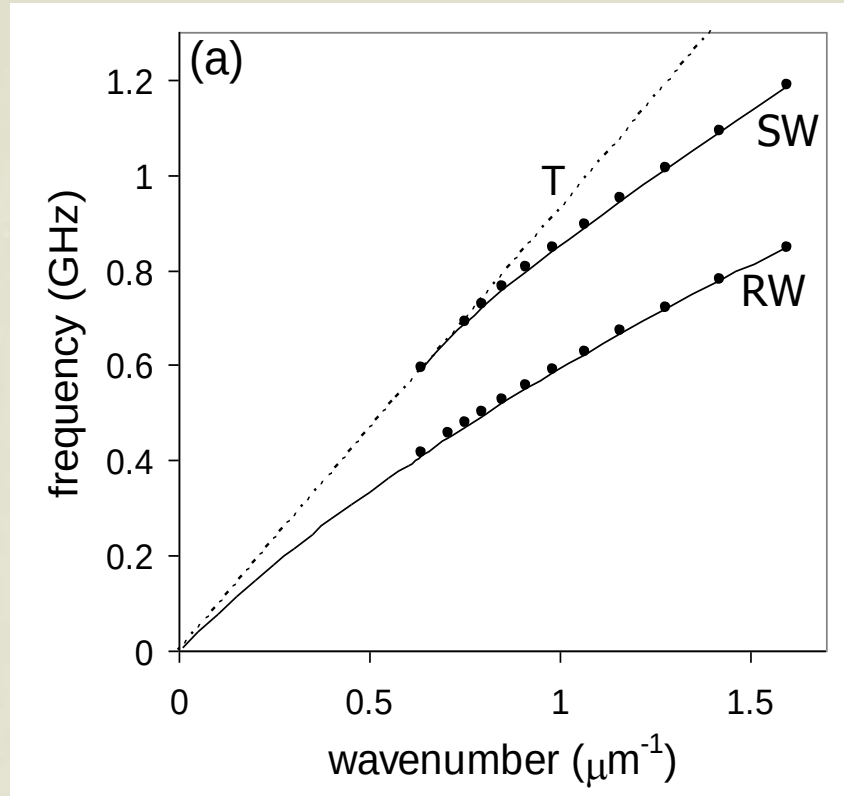
Laser Transient Grating Technique



SAW's are generated thermo-elastically by crossed laser pulses forming an interference pattern on the sample, the period of which defines the SAW wavelength, and can be varied. The SAW are detected by diffraction of a probe beam.

(b) A signal waveform obtained using the transient grating technique with k parallel to the bars. The Fourier spectrum of the acoustic oscillations of the surface is shown in the inset. Two modes are present here, which are the Rayleigh and first Sezawa modes.

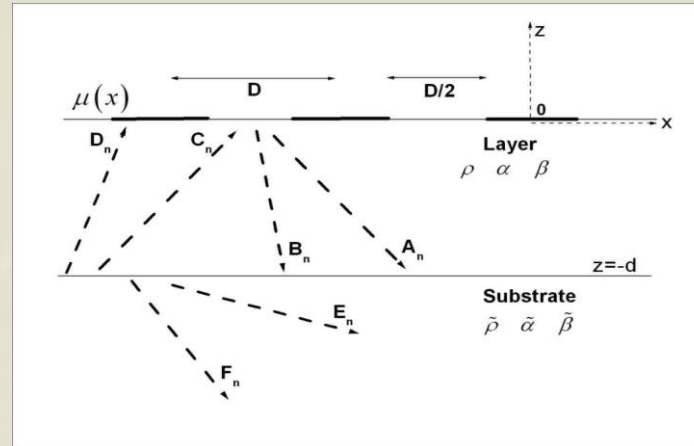
SAW Dispersion for Patterned Structure (Maznev et al.)



SAW dispersion curves (symbols) measured with wavevector (a) along copper bars and (b) perpendicular to the bars, i.e. in the direction of periodicity. Dotted lines correspond to bulk wave thresholds. Surface modes are “leaky” in the radiative zone above T. Solid lines in (a) were obtained by “effective medium” calculations. Solid lines in (b) correspond to the dispersion curves from (a) re-plotted vs. reduced wavenumber.

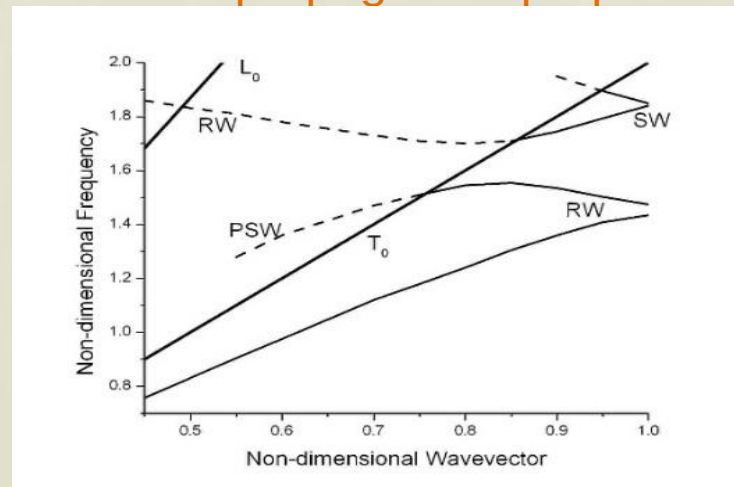
A.A. Maznev, Phys. Rev **B78**, 155323 (2008).

The Model



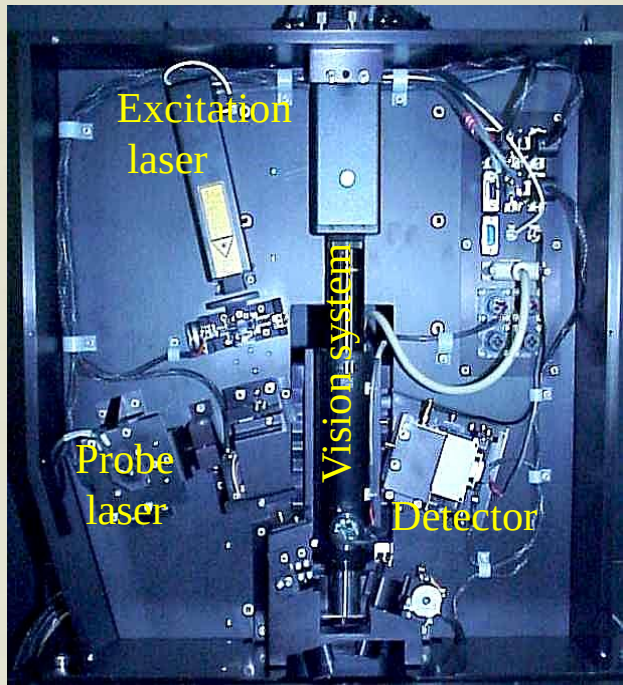
A model which captures the key attributes of the structures, and is able to account qualitatively for the main features in the observations posits a solid layer supported on a substrate. On top of the layer is a spatially periodic areal mass loading.

Calculated dispersion relation for propagation perpendicular to the bars.



Maznev and Every, J. Appl. Phys. **106**, 113531 (2009).

Compact and robust optics head integrated into a fully automated measurement station



50 cm

Acoustic wavelength range: 2-11 μm

Probe spot size: 30x17 μm

Measurement time: ~1s

SAW frequency range 0.1-1.2 GHz



Conclusions and Summary

- There are many different types of elastically anisotropic solids including crystals, textured polycrystalline solids, fiber composites and metamaterials, etc.
- Investigating these materials ultrasonically requires taking account of anisotropy.
- The wave equation for anisotropic solids and its solution has been presented, and concepts such as the distinction between phase and group velocity, the slowness surface and energy focusing have been discussed.
- Several applications have been discussed including phonon imaging, transmission acoustic microscopy of crystals, laser ultrasound scanning, metrology of single crystal turbine blades, the reflection and focusing of surface acoustic waves, surface Brillouin scattering and transient grating spectroscopy.